Passive UWB Beamforming: a N to M Compression Study

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***Abstract*—Recent works have demonstrated the feasibility of microwave imaging using compressive techniques, exempting the use the of active delay lines, phase shifters, or moving parts to achieve beamforming. With this method, waves are coded in a passive way by a compressive device to reduce the complexity of the transmitter and/or receiver chains of the telecommunication and radar systems requiring beamsteering. Since this technique is based on frequency diversity, the reduction of the compressive device's volume imposes a diminution of the amount of driven antennas. In this article, the improvement brought by simultaneous excitations of the compressive device is presented. Adapting a new mathematical formulation, it is shown that M inputs can send independent waveforms allowing the beamsteering of an N-elements antenna array, while maintaining $N > M$.**

I. Introduction

UWB radar techniques are used in a wide range of applications such as medical diagnosis and concealed objects detection. These techniques often take advantage of the possible transmission of microwaves in materials that are totally opaque in the optical domain. The capability of an imaging system to resolve targets depends of two parameters:

- The down range resolution $\delta_r = \frac{c}{2B}$
- The cross range resolution $\delta_{cr} = \min[\frac{c}{0.7079B}, \lambda_c]\frac{R}{L}$ [1]

with c the wave celerity, B the absolute bandwidth, λ_c the central wavelength, R the range of the focal point, and L the size of the radiating aperture. Thus, an high resolution imaging system requires the use of UWB hardwares connected to a large amount of antennas, representing an heavy burden regarding its cost and complexity. In the time domain, the aim of these active devices is to generate delays applied to each radiating elements and achieve beamsteering in the desired direction, as depicted in Fig. (1). It is worth noting that this principle can be applied in transmission as well as in reception.

In [2], a compressive technique has been presented to achieve beamforming with UWB signals in a passive way. This technique is based on the use of a passive multiplexer controlled by its input waveform, connected to an antenna array. This principle has been extended in [3], developing a compact compressive device and a new formulation of signal reconstruction. An other technique based on compressed sensing has been introduced in [4] and extended in [5], using a radiating device made of coupled resonators able to transmit signals in different directions of the space simultaneously depending on the frequency. Both these techniques are basically the same, exploiting the frequency diversity of a passive device to radiate orthogonal patterns at each frequency. A new study is presented in the next section, focused on the improvements

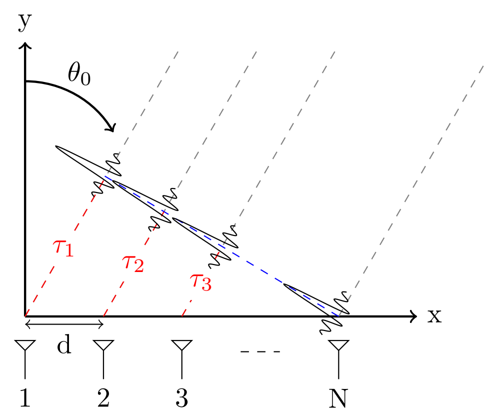


Fig. 1: Illustration of a time-domain beamforming: the delay applied to each radiated pulses depends on the direction of radiation and of the inter-element spacing.

brought by using several receiving chains connected to the compact compressive device introduced in [7]. The comparison between conventional and compressed beamforming is depicted to demonstrate the compressive capability of this technique and the modal diversity gain provided by using simultaneous measurements of independent coded waveforms.

II. Multiple-ports compressive beamforming

A. *Compressed acquisition theory*

The study presented in this article is based on a compression from a set of N signals received by an antenna array, and compressed into M signals after the propagation in a coding device, with $M < N$ (Fig. 2)

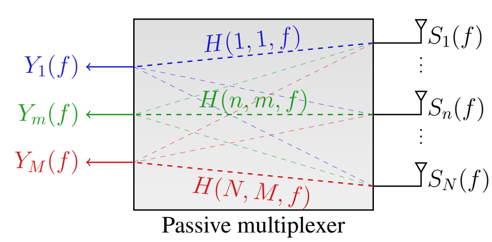


Fig. 2: Compression of a set of N signals received by an antenna array into M signals measured to compute digital beamforming.

The expression of $Y_k(f)$, the signal compressed at the device's output k is defined in eq. (1):

$$Y_k(f) = \sum_{n=1}^{N} H(n,k,f)\ S(n,f) \quad (1)$$

with $H(k,m,f)$ the compressive device transfer function between the ports k and m and $S(m,f)$ the signal received by the m^{th} antenna. This compression corresponds to a generalization of the experiments presented in [2][3][6][7], where the output was unique. In this article, the use of multiple outputs is studied to evaluate the gain for beamforming performed with a large number of antennas. Following the compressive sensing theory, the sensing matrix $[H]$ is sparse and must present a low level of correlation between its transfer functions, ensuring reduced inter-signals projections during the compressing process. Starting from the measured signals $[Y(f)]$, the received waveforms $[S(f)]$ are reconstructed to achieve digital beamforming using a pseudo-inversion of the matrix $[H(f)]$ at each frequency. Thus, the estimation of the signals $[S(f)]$ is:

$$[S(f)]_{rec} = [H(f)]^{+}\ [Y(f)] \quad (2)$$

$[H(f)]^{+}$ is a pseudo-inverse computed by truncated singular value decomposition (tSVD) of $[H(f)]$. In this purpose, the matrix $[H(f)]$ is factorized into three matrix for each frequency:

$$\underbrace{[H(f)]}_{N\times M} = \underbrace{[U(f)]}_{N\times N}\underbrace{[\Sigma(f)]}_{N\times M}\underbrace{[V(f)]^{\dagger}}_{M\times M} \quad (3)$$

with N the number of antennas and M the number of receiving chains. $[U(f)]$ and $[V(f)]$ are unitary matrices made of orthonormal set of basis, weighted by the singular values ordered in the quasi-diagonal matrix $[S(f)]$. The conditioning of this linear application is defined by the following equation:

$$cond([H(f]) = \frac{\sigma_1}{\sigma_N} \quad (4)$$

σ_1 and σ_N stand for the largest and the smallest singular values of $[H(f)]$. The condition number $cond([H(f])$ relates the accuracy of the solution given by the pseudo-inversion. An ideally conditioned problem would involve a condition number of 1, meaning that all the singular values have the same significance and are not sensitive to the inversion. For an ill-conditioned problem, the lowest singular values can be set to zero to improve the accuracy of the pseudo-inversion, expressed as :

$$[H(f)]^{+} = [V(f)][\Sigma(f)]_t^{-1}[U(f)]^{\dagger} \quad (5)$$

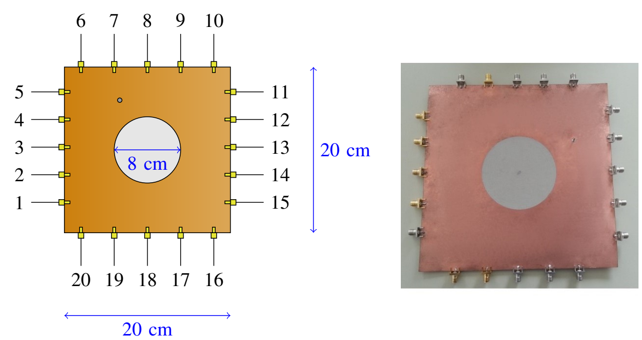
where $[\Sigma(f)]_t^{-1}$ is a quasi-diagonal matrix filled by $\{\frac{1}{\sigma_1},...,\frac{1}{\sigma_\beta},0,...,0\}$. All the remaining singular values must satisfy $\sigma > \beta$, with β a regularization parameter optimized empirically depending on the intrinsic conditioning of the problem. By computing this pseudo-inversion frequency by frequency, the inverse matrix $[H(f)]^{+}$ allows for a reconstruction of the waveforms received by the antenna array from the measured signals. Finally, radiation patterns $C(\theta,f)$ can be computed with the reconstructed signals $[S(f)]$. In the case of a far field approximation, the beamforming is computed in a conventional way by correlating the waveforms with plane waves :

$$C(\theta,f) = \sum_{n}^{N} S_{rec,n}(f)\ exp\left(j\frac{2\pi f}{c}d(n-1)\sin(\theta)\right) \quad (6)$$

with d the inter-antenna spacing and θ the beamsteering direction. In the next section, this principle is applied to a compact device connected to an antenna array to illustrate the beamforming improvement compared to conventional beamforming techniques applied with the same amount of receivers.

B. Application to a compressive device

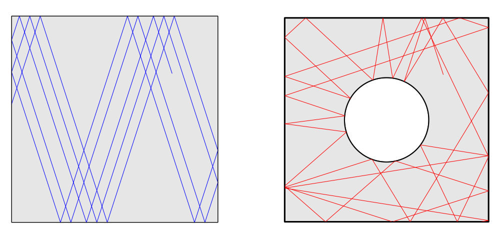
1) Study of the singular values: A $2D$ chaotic cavity introduced in [6][7] is used as a passive multiplexer (Fig. 3). This device is a microwave substrate *RT/duroid 6006* with a hole engraved on the its top conductor. 20 SMA ports are regularly connected on its edges, with small distance variations to break periodicity.



(a) Schematic representation (b) Realized prototype

Fig. 3: Planar compressive device connected to 20 SMA ports.

Despite its simple aspect, this microwave device presents very useful properties linked to the association of regular and convex boundaries delimiting the wave propagation [8][9]. These properties can be illustrated by replacing the field propagating in a cavity by its equivalent plane wave spectrum. The path followed by a plane wave can be represented as a ray bouncing on the cavity's boundary (Fig. 4).



(a) Regular cavity (b) Chaotic cavity

Fig. 4: Comparison between a regular and a chaotic cavity : propagation of a ray, starting from the same initial conditions

Starting from the same initial conditions, a ray in a chaotic cavity tends to fill the entire space after several reflexions compared to a regular cavity where the ray are focused in particular areas. Furthermore, a slight modification of the initial conditions of a ray in a chaotic cavity can have an important impact in the way it propagates, unlike the case of the regular cavity. Thus, even with a limited frequency diversity which represent the support of the signals compression, these properties help to decrease the correlation level between the device's transfer functions. The aim of this study is to demonstrate the interest of using several outputs simultaneously to achieve a better reconstruction of the signals when a large amount of antennas are connected to the compressive device. Thus, the transfer function between 16 inputs and 4 outputs are measured in the 2-4 GHz frequency range (300 points) to evaluate the singular value spectrum depending on the amount of connected outputs (Fig. 5).

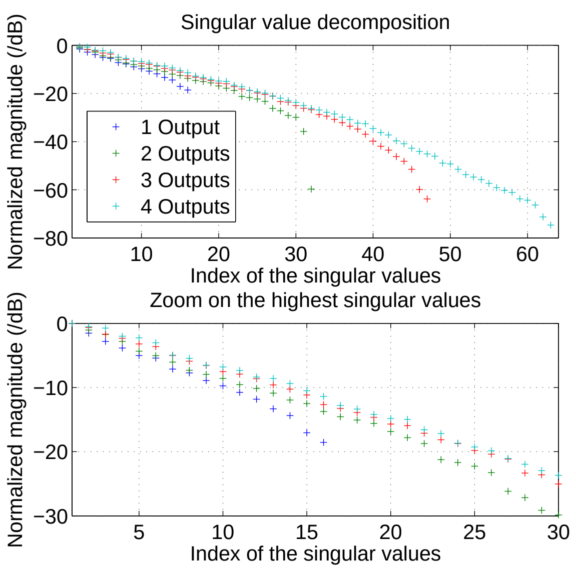


Fig. 5: Normalized magnitude of the singular values depending on the number of outputs connected to the compressive device.

More information is measured when new outputs are connected, leading to higher singular values and to an improved conditioning for a given regularization parameter.

2) Compressed beamforming: In conventional digital beamforming, as many receivers as antennas are required. With the same amount of receiving chains, the compressive beamforming can be performed with a larger number of antennas connected to the passive multiplexer. For this example, 16 antennas are connected to the chaotic device and compressed into 4 waveforms (Fig. 6). The beamforming capabilities of this system are compared with a conventional digital beamforming array made of four elements. These experiments are simulated by *Matlab* programs using the device's measured transfer functions in the frequency range 2-4 GHz. For both cases, the inter-antenna space is set to $d = 0.7 * \lambda_c = 7$ cm . The figure (7) depicts the results obtained for a beam-steering in the $25°$ direction, for a conventional system made of four elements and for a compressive system connected to 16 antennas. In both cases, the antennas are isotropically radiating and the mutual-coupling is not considered. The radiation patterns computed with eq. (6) are converted from frequency to time domain data using an inverse Fourier transform.

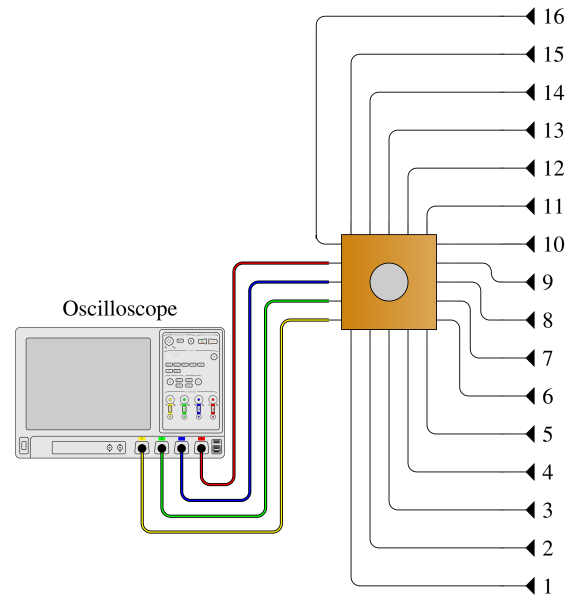


Fig. 6: Compressive beamforming system connected to 16 antennas, connected to four independent receiving chains. In the study of conventional beamforming, 4 antennas are directly connected to the receiver's outputs, while maintaining the same inter-antenna spacing.

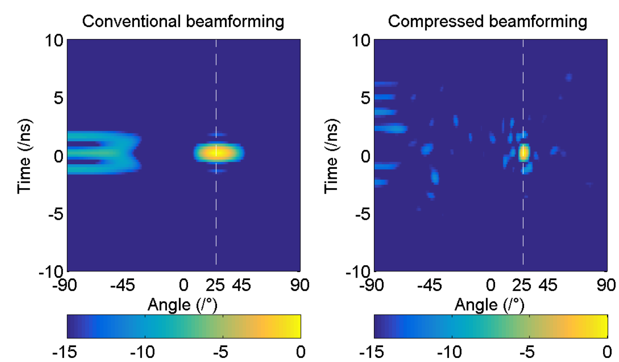


Fig. 7: Comparison of the time domain radiation patterns computed for a conventional beamforming with 4 antennas compared to a compressed beamforming of 16 antennas connected to four receivers via the compressive device. The beamsteering is performed in the $25°$ direction.

This figure shows that using the same number of receiving chains and inter-antenna space, the compressive technique allows for a thinner formed beam since the equivalent radiation aperture is wider and appropriately sampled. Furthermore, the grating lobes observed for the beamsteering with the conventional system are mitigate by the compressive device, due to a statistical cancellation of the off-axis frequency components [7]. The time required for the computation of the tSVD pseudo-inverse was less than 3.5×10^{-2} s and can be done only once and for all since the compressive device's transfer functions are stationary. The estimation of the received signals is calculated in less than 2×10^{-3} s. Radiation patterns are extracted at the focusing time $t = 0$ ns and presented in Fig. (8).

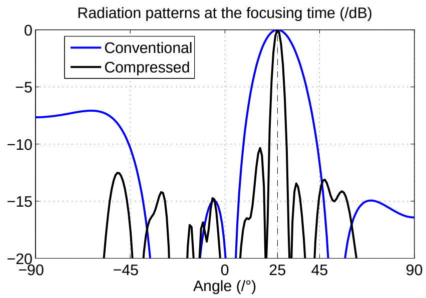


Fig. 8: Radiation patterns extracted at the focusing time from Fig. (7).

3) Simultaneous measurements and beamforming: The last part of this study is focused on the impact of the amount of connected outputs to achieve beamforming. An example is depicted in Fig. (9) to show the improvement provided by increasing the amount of connected outputs for a beamsteering in the $-60°$ direction.

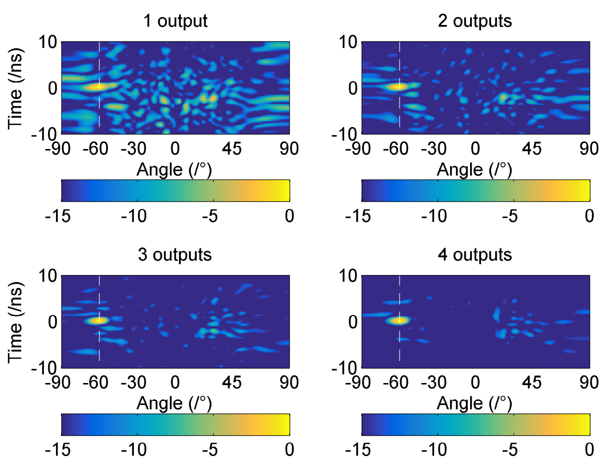


Fig. 9: Improvement of beamforming depending on the amount of connected outputs, for a beamsteering in the $-60°$ direction.

In this example, the level of noise tends to go down when the amount of connected outputs increase, meaning that new useful information are measured and decompressed to get better estimations of the waveforms received by the antennas. Thus, the peak signal-to-noise ratio (PSNR) is defined to determine the efficiency of the reconstruction for each beamsteering direction Φ, depending on the amount of connected outputs:

$$\mathrm{PSNR}(\Phi) = -10 \log_{10} \left(\frac{\sum_{\theta} \sum_{t} [|C_{\Phi}(\theta, t)| - |C_{\Phi,ideal}(\theta, t)|]}{n_{\theta} n_{t}} \right) \tag{7}$$

with $C_{\Phi}(\theta, t)$ the time domain radiation pattern computed with the compressive technique and $C_{\Phi,ideal}(\theta, t)$ the equivalent ideal radiation pattern calculated by plane wave projections over the position of the 16 antennas, both for a beamsteering in the direction Φ. Depending on the application, the trade-off between quality of the reconstruction and complexity of the associated hardware can be chosen. The computed results are gathered in Fig. (10).

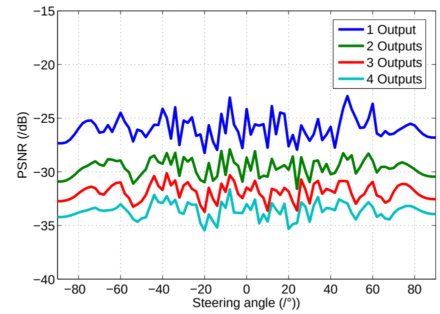


Fig. 10: Peak Signal to Noise ratio depending on the amount of connected outputs and of the steering angle.

As expected, the quality of the beamforming directly depends on the amount of measured signals from the compressive device, allowing for a better reconstruction of the received signals.

III. Conclusion

A compressive multiplexing technique has been studied in this paper, starting from previous works. The use of several outputs connected to a coding device has been shown efficient to increase the modal diversity required to achieve the compressed measurement of waveforms received by an antenna array made of 16 elements, over 4 receiving chains. Thus, the improvement compared to conventional digital beamforming using the same number of receivers and the same inter-element spacing has demonstrated the possible implementation of this technique to reduce the hardware complexity of the UWB radars.

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