Alternative quantum circuit implementation for 2D electromagnetic wave simulation with quasi-PEC modeling

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Nicolas Bui[1], Alain Reineix[2], Christophe Guiffaut[2]

[1]Université de Limoges, XLIM, UMR 7252, Limoges, Nouvelle-Aquitaine, 87000, Nicolas.Bui@xlim.fr, France
[2]CNRS, XLIM, UMR 7252, Limoges, Nouvelle-Aquitaine, 87000, Alain.Reineix@xlim.fr, France

***Index Terms* — Quantum Computing, Maxwell solver, PEC modeling.**

ABSTRACT

Numerical simulation may benefits from quantum computing (QC) potential to solve particular problems faster than its classical counterpart. This motivates recent works [1, 2, 3] to solve the wave equation with a quantum computer. Here, we consider 2D wave propagation in free space based on an alternative implementation of the quantum algorithm developed in [2, 3]. We also propose a way to model quasi-perfect electrical conductor (PEC). An example of a plane wave reflection is given, with QC on a simulator.

I. MAXWELL-DIRAC FORMULATION AND PEC MODELING

Following [2, 3], the Maxwell-Dirac equations are obtained by introducing the Riemann-Silberstein vectors in the Maxwell equations:

$$F^{\pm} = \sqrt{\epsilon}E \pm iB\mu^{-1/2};\ \psi^{\pm} = (-F_x^{\pm} \pm F_y^{\pm}, F_z^{\pm}, F_z^{\pm}, F_x^{\pm} \pm F_y^{\pm}) \quad (1)$$

Then in a homogeneous medium, the Maxwell equation can be seen as a Dirac equation with mass *m=0* and particular choices of Pauli matrix a:

$$\partial_t \psi^{\pm} = c \sum_{i=1}^{3} a \otimes \sigma_i\, \partial_i \psi^{\pm} + imc^2 \bar{h}^{-1} b \otimes I_2 \psi^{\pm} \quad (2)$$

The quantum circuit associated to this time evolution is then easier to build since it is known for Pauli matrices, and the displacement operators (oriented) are SWAP or CNOT operations. Naming such operators C and S respectively, the time evolution scheme is finally [2, 3](omitting phase $e^{-\delta_t^2}$):

$$\psi(t + \delta_t) = S_- C S_+ C^* S_+ C S_- C^* M^* S_+ C^* S_- C S_- C^* S_+ C M \psi(t) \quad (3)$$

M is the identity matrix or the quasi-PEC matrix as the following product:

$$\mathrm{MyMx} = \begin{pmatrix} i\sin(\theta) & 0 & 0 & \cos(\theta) \\ 0 & i\sin(\theta) & -\cos(\theta) & 0 \\ 0 & -\cos(\theta) & i\sin(\theta) & 0 \\ \cos(\theta) & 0 & 0 & i\sin(\theta) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

The transmission coefficient is -120dB and -60 dB for *My* and *Mx* respectively.

II. IMPLEMENTATION AND COMPUTATION ON SIMULATOR

Yepez [3] proposes a quantum circuit implementation for the Schrödinger equation, which can also be used here. This implementation uses a number of complexes which is much higher than the number of components of ψ. Instead, our implementation uses a number of complexes equal to the number of components of ψ, thus reducing significantly the required number of qubits. This implementation follows the data structure used in [4] for the Transmission Line Method.

Spatial positions are encoded using the Gray code. Displacement operators are multiple CNOT circuits. Applying this on the whole domain yields periodic boundary conditions. The circuit to retrieve the electric field from (1) uses CNOT and Controlled H gates.

This circuit was implemented on IBM simulator (Qiskit). Fig. 1 is a plane wave injected at the middle (x=16) that has just reflected on an infinite PEC wall at x=5 Note that quantum computing enforces normalization on the whole space.

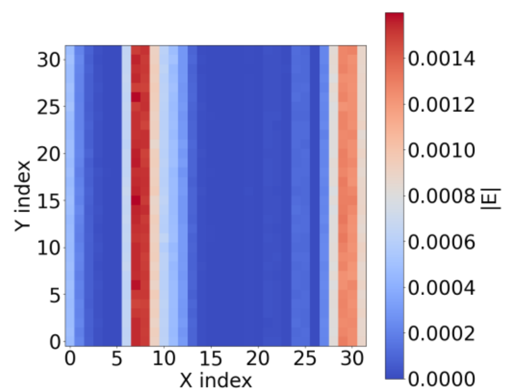

Fig. 1. Space-normalized electric field modulus.

III. REFERENCES

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